Dept. of Mathematics,

M.Sc. Sem II, Paper- IX

Topology

Connectedness - Let X be a topological space. Then X is said to be dis-connected if there exist two (Non-Epty, Disjoint opensets) G and H of X such that $X = G \cup H$.

Now, $X = G \cup H$ and G and H are disjoint

$\Rightarrow G \cap H = \phi$

So, $G = H^c$ and $H = G^c$

If G and H are open, they are closed also.

Thus, a topological space X is said to be connected if X is not dis connected.

Theorem :- A subset Y of a topological space X is disconnected iff Y is contained in the union of two open subsets G, H of X whose intersections with Y are non-empty and disjoint.

Proof :- we know
$Y \cap (G \cup H) = (Y \cap G) \cup (Y \cap H)$

$\therefore Y \subseteq G \cup H \Leftrightarrow Y = (Y \cap G) \cup (Y \cap H)$ —— (I)

Let Y be disconnected.

$\Rightarrow Y$ is union of two non-empty disjoint open sets, sets which are open for induced topology.

Since, An open set of the induced topology is of the form $Y \cap G$ where G is open in X.

$\Rightarrow$ $\exists$ two open subsets G, H of X such that $Y = (Y \cap G) \cup (Y \cap H)$ —— (2)
such that $Y \cap G$ and $Y \cap H$ are non-empty and disjoint.

Eqn. (2) $\Rightarrow Y \subseteq G \cup H$. proved.

To prove Y is disconnected

From (1)

$$Y = (Y \cap G) \cup (Y \cap H) \quad \text{—— (3)}$$

$\Rightarrow$ (3) is a disconnection of Y

$\Rightarrow$ Y is disconnected.

* A topology on a set X is a collection $\mathcal{J}$ of subsets of X having the following properties:

(i) ϕ and X are in $\mathcal{J}$.

(ii) The union of the elements of any subcollection of $\mathcal{J}$ is in $\mathcal{J}$.

(iii) The intersection of the elements of any finite subcollection of $\mathcal{J}$ is in $\mathcal{J}$.

A set X for which a topology $\mathcal{J}$ has been specified is called a topological space.
If X is a topological space with topology $\mathcal{J}$, we say that a subset U of X is an open set of X. If U belongs to the collection $\mathcal{J}$.
If X is any set, the collection of all subsets of X is a topology on X, it is called the discrete topology.